Using the Color Calculator

The calculator on your phone can easily convert fractions to their decimal equivalents. Many calculators display results to 10 decimal places, which certainly seems like a lot. For a fraction like $\frac{1}{3}$ with its infinitely repeating 3s or $\frac{1}{4}$ with its trails of zeroes, 10 decimal places more than suffices. But for a fraction like $\frac{1}{17}$ that doesn't repeat until its sixteenth decimal place, a display with 10 decimal places is not enough.

The Color Calculator app explodes the 10 decimal place limit by allowing you to view, in table form, 225 digits to the right of the decimal point for any fraction. You can choose how many digits appear in each row of the table (the *table width*) to make it easier to spot patterns in the digits. The Color Calculator also associates each digit in the decimal expansion of a fraction with a unique color, allowing you to hunt for numerical patterns that the string of numbers don't make obvious. You can even hide all the digits and just view their colors.

Here are some activity suggestions for the app.

1. Experiment with different fractions of your own choosing. What do you notice?
2. Look for patterns in the rows, columns, and diagonals of the decimal representations of your fractions. Change the *table width* to change the patterns and to create new ones.
3. Turn on *Hide Digits* to view just the color patterns without getting distracted by the digits.
4. Find fractions whose decimal representations eventually terminate and end in all zeros. How can you tell without checking whether a fraction has this property?
5. If the decimal representation of a fraction contains 10 zeros in a row, does that imply the decimal representation terminates?
6. Find fractions whose decimal representations consist of just one repeating digit (e.g., 0.33333…) Can you find a fraction for every digit 1 through 9?
7. Find fractions whose decimal representations have really long sequences of digits that eventually repeat. Do you think there are fractions whose decimal representations never repeat?
8. What do you notice about the decimal representations of the fractions $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$?
9. What do you notice about the decimal representations of fractions with a denominator of 13? How do these patterns differ from the ones you noticed for fractions with a denominator of 7?