THE MYSTERY OF POLYGON FLATS 1 (page 1 of 2)

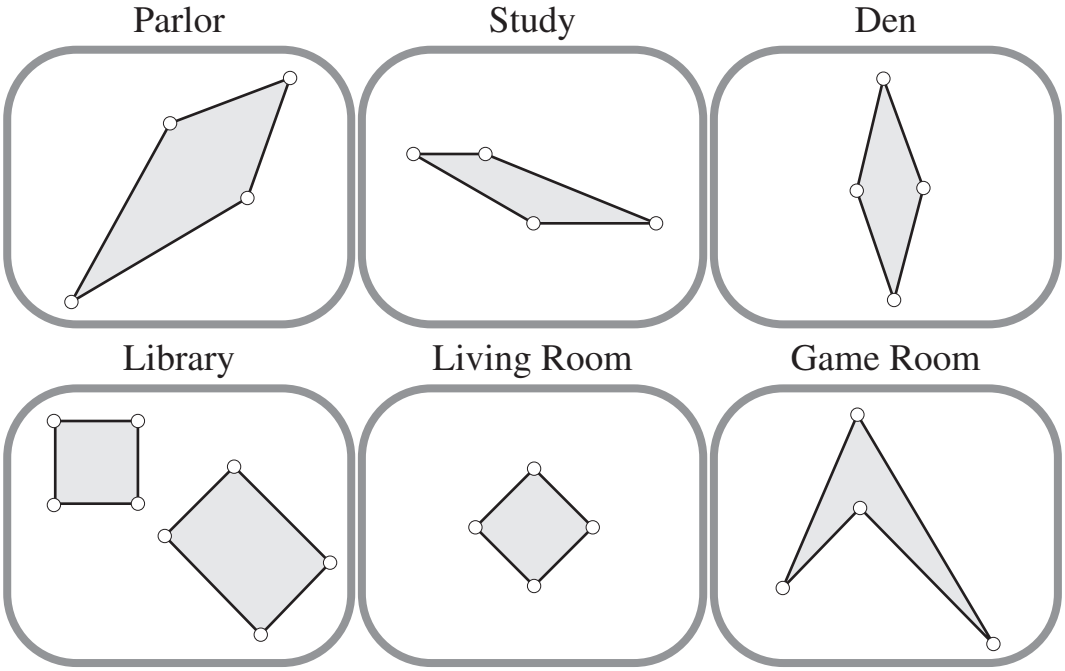
Quadrilateral Explorations 5–6

Measured Quad Makers.gsp

Famous female detective Shirley Lock-Holmes is in the two-dimensional town of Polygon Flats investigating a theft in Quadrilateral Mansion. The seven people who live in the mansion are listed below. These people can change their shape and size but only to a shape that can be made by their Shape Maker.

Sudha Square (played by the Square Maker)
Rectangle Rick (played by the Rectangle Maker)
Kaneisha Kite (played by the Kite Maker)
Parallelogram Pete (played by the Parallelogram Maker)
Trapezoid Tracy (played by the Trapezoid Maker)
Ricardo Rhombus (played by the Rhombus Maker)
Quentin Quadrilateral (played by the Quadrilateral Maker)

When the theft occurred, the people in the mansion looked as follows in the mansion's security cameras. Examine the evidence, including the camera pictures below and clue on the next page, and figure out who committed the theft.



Shape Makers ©2012 by Key Curriculum Press

THE MYSTERY OF POLYGON FLATS 1 (page 2 of 2)

Clue
1. The theft occurred in the study.

Who is the thief? _____

Write an argument that proves "beyond the shadow of doubt" that the person you have accused of being the thief is guilty.

_____

_____

_____

_____

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_____

THE MYSTERY OF POLYGON FLATS 2 (page 1 of 2)

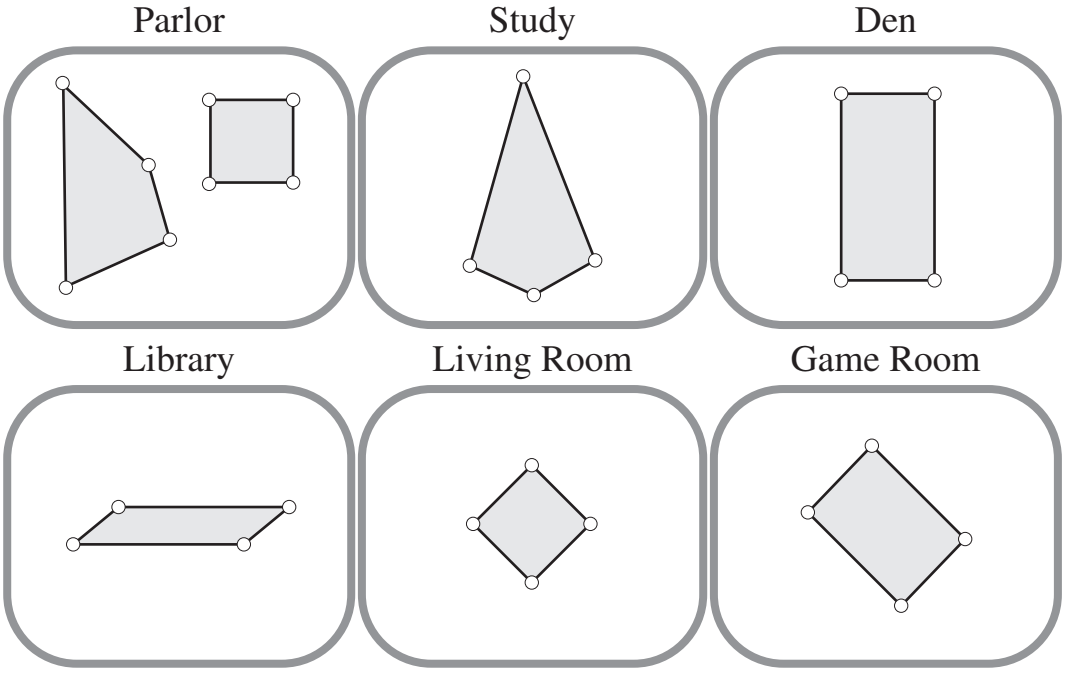
Quadrilateral Explorations 5–6

Measured Quad Makers.gsp

Famous detective Shirley Lock-Holmes is in the two-dimensional town of Polygon Flats investigating a theft in Quadrilateral Mansion. The seven people who live in the mansion are listed below. These people can change their shape and size but only to a shape that can be made by their Shape Maker.

Sudha Square (played by the Square Maker)
Rectangle Rick (played by the Rectangle Maker)
Kaneisha Kite (played by the Kite Maker)
Parallelogram Pete (played by the Parallelogram Maker)
Trapezoid Tracy (played by the Trapezoid Maker)
Ricardo Rhombus (played by the Rhombus Maker)
Quentin Quadrilateral (played by the Quadrilateral Maker)

When the theft occurred, the people in the mansion looked as follows in the mansion's security cameras. Examine the evidence, including the camera pictures below and clues on the next page, and figure out who committed the theft.



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THE MYSTERY OF POLYGON FLATS 2 (page 2 of 2)

Clues
1. The theft occurred in the library. **2. The thief sometimes has four unequal sides.**

Who is the thief? _____

Write an argument that proves "beyond the shadow of doubt" that the person you have accused of being the thief is guilty.

_____

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THE MYSTERY OF POLYGON FLATS 3 (page 1 of 2)

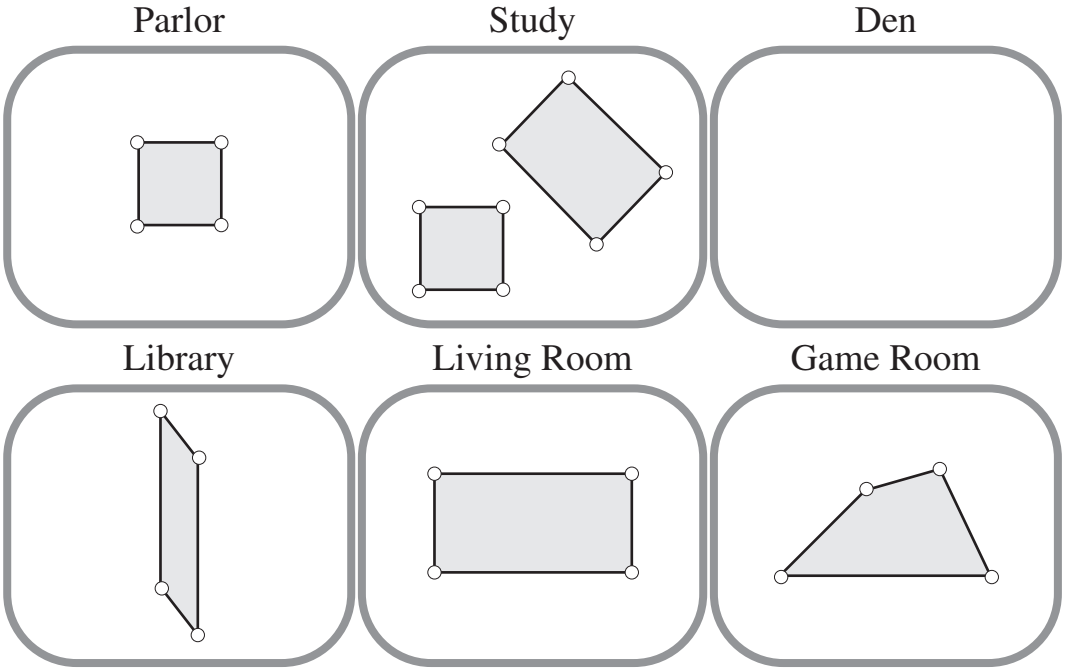
Quadrilateral Explorations 5–6

Measured Quad Makers.gsp

Famous detective Shirley Lock-Holmes is in the two-dimensional town of Polygon Flats investigating a theft in Quadrilateral Mansion. The seven people who live in the mansion are listed below. These people can change their shape and size but only to a shape that can be made by their Shape Maker.

Sudha Square (played by the Square Maker)
Rectangle Rick (played by the Rectangle Maker)
Kaneisha Kite (played by the Kite Maker)
Parallelogram Pete (played by the Parallelogram Maker)
Trapezoid Tracy (played by the Trapezoid Maker)
Ricardo Rhombus (played by the Rhombus Maker)
Quentin Quadrilateral (played by the Quadrilateral Maker)

When the theft occurred, the people in the mansion looked as follows in the mansion's security cameras. Examine the evidence, including the camera pictures below and clues on the next page, and figure out who committed the theft.



THE MYSTERY OF POLYGON FLATS 3 (page 2 of 2)

Clues
1. The theft occurred in the parlor. 2. People who are always right are always in the same room. 3. A well-balanced person (who always has at least two lines of symmetry) was out of the mansion at the time of the theft.

Who is the thief? _____

Write an argument that proves "beyond the shadow of doubt" that the person you have accused of being the thief is guilty.

_____

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THE MYSTERY OF POLYGON FLATS 4 (page 1 of 2)

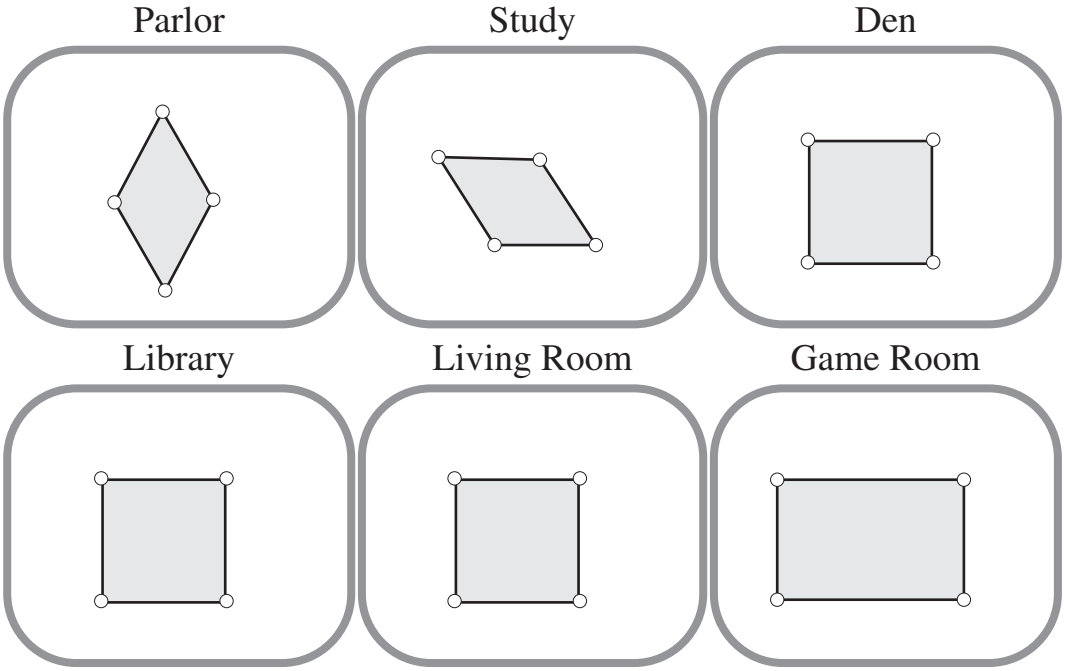
Quadrilateral Explorations 5–6

Measured Quad Makers.gsp

Famous detective Shirley Lock-Holmes is in the two-dimensional town of Polygon Flats investigating a theft in Quadrilateral Mansion. The seven people who live in the mansion are listed below. These people can change their shape and size but only to a shape that can be made by their Shape Maker.

Sudha Square (played by the Square Maker)
Rectangle Rick (played by the Rectangle Maker)
Kaneisha Kite (played by the Kite Maker)
Parallelogram Pete (played by the Parallelogram Maker)
Trapezoid Tracy (played by the Trapezoid Maker)
Ricardo Rhombus (played by the Rhombus Maker)
Quentin Quadrilateral (played by the Quadrilateral Maker)

When the theft occurred, the people in the mansion looked as follows in the mansion's security cameras. Examine the evidence, including the camera pictures below and clues on the next page, and figure out who committed the theft.



THE MYSTERY OF POLYGON FLATS 4 (page 2 of 2)

Clues
1. The theft occurred in the game room. 2. People in the parlor and game room always have at least two equal sides. 3. At the time of the theft, one person was at the airport, unstrung because there was not enough wind to fly. 4. The person in the study always has at least two lines of symmetry but sometimes has exactly two.

Who is the thief? _____

Write an argument that proves "beyond the shadow of doubt" that the person you have accused of being the thief is guilty.

_____

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MAKE YOUR OWN MYSTERY OF POLYGON FLATS (page 1 of 4)

Quadrilateral Explorations 5–6

Your Own Polygon Flats.gsp

You have been invited by the *Polygonal Press* to write a mystery puzzle for next week's newspaper. Because of your extensive experience with the Shape Makers and your logical reasoning abilities, the editors feel that you can write a Polygon Flats puzzle that will throw a curve to the weekly readers.

Your task is to write Your Own Polygon Flats Mystery.

Directions

1. Open the sketch **Your Own Polygon Flats.gsp** on the computer. On the screen, you will see the seven Shape Makers labeled with their names.
2. You must first decide which Shape Maker has committed the crime and in what room the crime was committed. You must then decide which room each of the other Shape Makers was in when the crime was committed. It is possible for two Shape Makers to be in the same room. On a piece of scrap paper, record the room location for each Shape Maker and who committed the theft.
3. Now you are ready to use the computer to move each Shape Maker into the room where you have decided it should go. To do this, move the Shape Maker into the room where it belongs. Now change the Shape Maker into a shape of your choosing. For example, if the Parallelogram Maker is in the study, you may want it to make a parallelogram that other Shape Makers could also make. If the Parallelogram Maker is in the shape of a rectangle, then the reader won't know whether the Rectangle Maker is in the study or the Parallelogram Maker is in the study. This will make the puzzle more difficult to solve.
4. You should now write clues that will help readers find out which Shape Maker committed the crime. Record the clues on page 4 of the "Make Your Own Mystery of Polygon Flats" student sheet. Also on this sheet, draw what the Shape Makers in each room look like (without their names, of course).
5. After you have completed all the details needed to solve the puzzle, you are ready to hide the names of each Shape Maker. To do this, click on the *Hide names* button.

Shape Makers ©2012 by Key Curriculum Press

MAKE YOUR OWN MYSTERY OF POLYGON FLATS (page 2 of 4)

6. (Optional) Your teacher may want you to save your work. To do this, go to the File menu and choose **Save As**. A dialog box will appear asking you to save your puzzle. Name your sketch according to your teacher's instructions.
7. Now you are ready for a test run. Invite another pair of students to solve your puzzle. Have them come over to your computer. Show them the clues that you have written and your mystery computer screen. Have them try to solve your mystery. Keep your answer hidden. At this time they are not allowed to manipulate the Shape Makers on the screen. Silently observe how they solve your mystery.
8. When the students have solved your puzzle, they can check it on the screen. To do this, they click on the *Show names* button.

Revising Your Mystery

1. The students who solve your mystery should explain to you their solution and the strategy they used to solve your mystery. If they cannot solve your mystery, they should explain why they were unable to do so.
2. When these students are finished solving your mystery, you will go to their computer and solve their mystery.
3. There may be a class discussion when all students have solved one mystery. This is an opportunity for any groups who did not agree on a solution to come to a resolution with the help of the rest of the class.
4. You may now go back to your computer and make revisions to your mystery. You may wish to make changes
   - if it was unsolvable by the other pair of students;
   - if your mystery did not make sense to them and you need to change some clues or shapes.
5. After making your revisions, find *another* pair of students to solve your mystery. You attempt to solve their mystery. Again, talk about the strategies you used and about any other revisions that might be needed. Was your first revision successful?
6. Make any further revisions that are needed.
7. Your class may wish to put together a book with all of the students' mysteries.

MAKE YOUR OWN MYSTERY OF POLYGON FLATS (page 3 of 4)

Quadrilateral Explorations 5–6

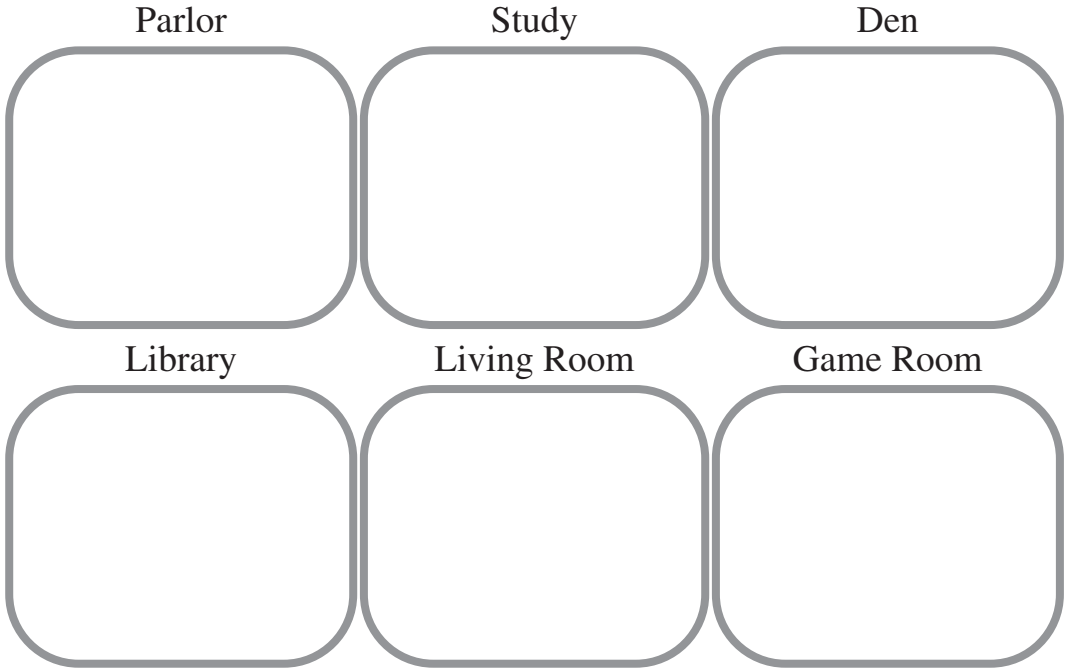
Your Own Polygon Flats.gsp

Written by _____

Famous detective Shirley Lock-Holmes is in the two-dimensional town of Polygon Flats investigating a theft in Quadrilateral Mansion. The seven people who live in the mansion are listed below. These people can change their shape and size but only to a shape that can be made by their Shape Maker.

Sudha Square (played by the Square Maker)
Rectangle Rick (played by the Rectangle Maker)
Kaneisha Kite (played by the Kite Maker)
Parallelogram Pete (played by the Parallelogram Maker)
Trapezoid Tracy (played by the Trapezoid Maker)
Ricardo Rhombus (played by the Rhombus Maker)
Quentin Quadrilateral (played by the Quadrilateral Maker)

When the theft occurred, here's how the people in the mansion looked in the mansion's security cameras. Examine the evidence, including the drawings on this student sheet, the clues on the next student sheet, and the camera pictures on the computer. Figure out who committed the theft.



MAKE YOUR OWN MYSTERY OF POLYGON FLATS (page 4 of 4)

Clues

Who is the thief? _____

Write an argument that proves "beyond the shadow of doubt" that the person you have accused of being the thief is guilty.

_____

_____

_____

_____

_____

_____

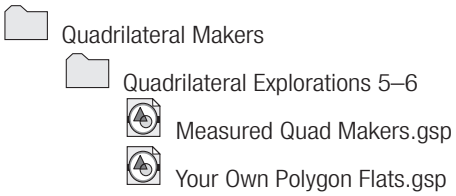
Shape Maker Mysteries

Summary

In the context of a series of "whodunnit" mystery stories, students are to discover which unlabeled Shape Maker "committed a theft." They are given two types of clues to help them solve the mysteries: (1) they are shown unlabeled pictures of Shape Makers, and (2) they are given verbal statements related to properties of the Shape Makers.

Mathematical Objectives

These activities are intended to sharpen students' knowledge of the properties of the Shape Makers, as well as to encourage the type of logical deduction often required in mathematics.



Required Materials

Session	Student sheet	SS#	Geometer's Sketchpad sketch
1–3	The Mystery of Polygon Flats 1–4	25–32	Measured Quad Makers.gsp
4 and 5	Make Your Own Mystery of Polygon Flats	33–36	Your Own Polygon Flats.gsp

Activity: The Mystery of Polygon Flats

Use Student Sheets 25–32 and this sketch:

Quadrilateral Explorations 5–6

Measured Quad Makers.gsp

Class Discussion

➦ Distribute the "The Mystery of Polygon Flats 1" student sheets to the class, and explain the task:

Famous female detective Shirley Lock-Holmes is in the two-dimensional town of Polygon Flats investigating a theft in Quadrilateral Mansion—someone stole the owner's special cream-filled cupcakes. Seven people live in the mansion. These people can change their shape and size but only to a shape that can be made by their Shape Maker. They are

Sudha Square	*(played by the Square Maker)*
Rectangle Rick	*(played by the Rectangle Maker)*
Kaneisha Kite	*(played by the Kite Maker)*
Parallelogram Pete	*(played by the Parallelogram Maker)*
Trapezoid Tracy	*(played by the Trapezoid Maker)*
Ricardo Rhombus	*(played by the Rhombus Maker)*
Quentin Quadrilateral	*(played by the Quadrilateral Maker)*

When the theft occurred, the people in the mansion looked like this in the TV room monitors [point to the graphics on the student sheet]. Your job is to determine who committed the theft. You must prove your answer is correct because whoever you accuse of the crime will have a good lawyer.

You are given two types of clues: (1) You are shown what the people (Shape Makers) looked like and what rooms they were in at the time of the theft; (2) you are provided important information about people in the mansion. There are several mysteries to solve, each more difficult to prove than the one before.

Note that for most of the mysteries, you will not be able to determine the whereabouts of each character. You are given only enough information to determine the identity of the thief.

You can use the Shape Makers on the computers as well as your notes and previous student sheets to help you solve the mysteries. Good luck!

Students must use logical reasoning and their knowledge about the Shape Makers to solve the mysteries; then they must write out a complete argument that proves their answer correct. Example solutions are given later in this Exploration, and illustrations of students' reasoning on these tasks are given in "Teaching Note: Right Answers, Incomplete Reasons," on page 78.

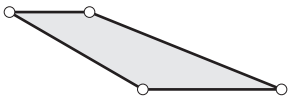
Students Work in Pairs

➦ Have students use the Measured Quadrilateral Makers on the computer to help them figure out answers for Mystery 1.

Use of the Shape Makers makes the problems accessible—most students who have difficulty thinking logically in abstract contexts seem to be able to make inferences based on their manipulations of the Shape Makers.

➦ As you're interacting with pairs of students, if you see students with incorrect answers, you might challenge them in some way:

Are you sure that the Quadrilateral Maker is the only Shape Maker that can make this figure?



The reasoning required in solving these mysteries is similar to that required in forming geometric proofs. For a completely rigorous solution, students must use logical deduction and justify each step in their arguments. However, keep in mind that many middle school and junior high students have difficulty reasoning so abstractly. Do not make students whose arguments are not completely rigorous feel that their answers are incorrect; they are passing through a natural stage in the development of the kind of abstract reasoning we are trying to encourage. Through class discussions of student solutions, students will cooperatively establish standards of rigor that they feel comfortable with. Your task is to encourage them to make their standards as rigorous as they can.

Class Discussion

➦ Have several students present their solutions for Mystery 1, step by step.

Other students should ask questions about statements they don't understand and challenge any statements they think are false. Although there is only one correct answer to who committed the crime, there may be several valid ways to deduce who the thief is. Even so, students may offer incorrect or incomplete solutions like some of those illustrated in "Teaching Note: Right Answers, Incomplete Reasons," on page 78.

In the class discussion, encourage students to challenge each other's arguments. If no other student challenges an incorrect or incomplete argument in the class discussion, you might challenge it, as illustrated in "Teaching Note: Right Answers, Incomplete Reasons."

Students Work in Pairs

➦ Have students use the Measured Quadrilateral Makers on the computer to help them figure out answers for Mysteries 2, 3, and 4.

When all students have finished Mystery 2, you can have a class discussion on it, then let students go back to working on the other mysteries.

Example Solutions

Mystery 1. Only the Trapezoid Maker and the Quadrilateral Maker can make the shape in the study (so only they could have committed the theft). But the Quadrilateral Maker is the only Shape Maker that can make the shape in the game room. Thus, Trapezoid Tracy must have committed the theft.

Mystery 2. The only Shape Makers that can have four unequal sides are the Quadrilateral Maker and the Trapezoid Maker, so clue 2 indicates that these two are our only possible suspects. Because the theft occurred in the library and the Quadrilateral Maker must be in the parlor (because it is the only Shape Maker that can make the nonsquare shape), Trapezoid Tracy must be the thief.

Mystery 3. The clues in this mystery are less straightforward. Although clue 1 tells us the room the theft occurred in, it does not eliminate any suspects, because all the Shape Makers can make a square. Clue 2 means that the Square Maker and the Rectangle Maker are in the same room, which, according to the picture, must be the study. Clue 3 tells us that the Square, Rectangle, or Rhombus Maker was out of the house, because these are the only Shape Makers that always have at least two lines of symmetry. But because we know that the Square and Rectangle Makers were in the house, clue 3 tells us that the Rhombus Maker was out of the house. The only shapes in the picture that can be made by the Kite Maker are the squares. But the square in the study is the Square Maker. Thus, the square in the parlor must be the Kite Maker, so Kaneisha Kite must be the thief. (You can also reason that the Quadrilateral Maker must be the shape in the game room and that the only Shape Makers left that could make the shapes in the library and living room are the Parallelogram Maker and Trapezoid Maker. Thus, by the process of elimination, the Kite Maker must be in the parlor.)

Mystery 4. One "proof" that Rectangle Rick committed the theft goes as follows: Because the theft occurred in the game room, the thief must be able to make the shape shown in this room. Sudha Square, Kaneisha Kite, and Ricardo Rhombus cannot make this shape. Although Quentin Quadrilateral and Trapezoid Tracy can make this shape, they are eliminated as suspects because they don't always have at least two equal sides (clue 2). Thus, we have eliminated all but two suspects—Parallelogram Pete and Rectangle Rick. To implicate Rick, we must show that Pete is not in the game room. The shape in the parlor could only be made by Kaneisha Kite, Parallelogram Pete, Trapezoid Tracy, Ricardo Rhombus, or Quentin Quadrilateral. Clue 2 eliminates Tracy and Quentin. Clue 3 indicates that Kaneisha Kite was at the airport, so she couldn't be in the parlor. The only Shape Makers that always have at least two lines of symmetry are the Square, Rectangle, and Rhombus Makers, so clue 4 indicates that Ricardo Rhombus is in the study and thus couldn't be in the parlor. That means that Pete is in the parlor. Therefore, Rick is the culprit.

SESSIONS 4 AND 5

Activity: Make Your Own Mystery of Polygon Flats

Use Student Sheets 33–36 and this sketch:

Quadrilateral Explorations 5–6

Your Own Polygon Flats.gsp

Students Work in Pairs

➦ Have student pairs make their own mysteries for Polygon Flats (see the directions on page 1 of the "Make Your Own Mystery of Polygon Flats" student sheets).

This is a task that students get quite excited about, but it is also difficult to do correctly.

To show where the Shape Maker personalities were during the theft, students put the Shape Makers in the sketch **Your Own Polygon Flats.gsp** into rooms, then click on the *Hide names* button to hide the names of the Shape Makers. They then write their own mystery clues.

➦ After students in each pair have made their mystery, ask them to try to solve it themselves. This helps them check their logic and clues and make appropriate revisions.

➦ Have each student pair give their mystery to another pair to solve.

For instance, Pair 1 should have Pair 2 try to solve Pair 1's mystery. Pair 1 should silently observe Pair 2 trying to solve Pair 1's mystery so that Pair 1 students understand any difficulties that Pair 2 has or any errors that Pair 1 might have made in logic or in writing clues.

After Pair 2 has deduced a solution for Pair 1's mystery, they should click on the *Show names* button to see if their answer is correct. The two pairs should then talk about any errors that occurred in either the solving or the writing of the mystery. They should then switch roles, with Pair 1 trying to solve Pair 2's mystery.

If students find that they made mistakes in their constructions of mysteries, they should revise them, then test them again by sharing them with another pair of students.

Class Discussion

➦ After all student pairs have traded mysteries, have a class discussion of some of the mysteries and solutions.

There may be disagreements over whether the given clues are sufficient or whether there is more than one solution. Have the class arbitrate these disagreements.

When all students have completed their revisions and are satisfied that they have made good mysteries, the class might put together a book of mysteries. You (or the students) might even want to copy student pictures from the **Your Own Polygon Flats.gsp** sketches into a word processor document containing their clues. (Or you could print these pictures and tape them onto Student Sheet 35.)

Homework

➦ Distribute the "Relating Shape Makers" student sheets to the class. Students are asked questions that encourage them to think about interrelationships between various Shape Makers. They predict answers for homework, then on the following day test their predictions using the Shape Makers.

If you have students write their predictions in one color and the answers they find in pairs at the computer on the next day in another, you can collect these sheets and use them to assess your students' individual thinking.

Right Answers, Incomplete Reasons

You will find many students who find correct answers to the mysteries but present incorrect or logically incomplete arguments. Students giving incomplete solutions should be encouraged to elaborate their proofs. Questions such as "How did you know that this Shape Maker didn't make this shape?" can be very helpful in encouraging students to further justify their answers.

Students' reasoning can also be flawed because they have mistaken notions about what shapes the Shape Makers can make. For instance, one student thought that the Quadrilateral Maker had to have at least two equal sides—thus, clue 2 in Mystery 4 did not eliminate this Shape Maker as a suspect for this student.

Another way to help students who are stuck is to ask them to tell you what they are thinking: "What does this clue tell you? What Shape Makers could be in this room?"

Several levels of sophistication will be seen in students' solutions, as illustrated here.

Mystery 2, Darla. "I think it's the Quadrilateral Maker because we put all the other Shape Makers in rooms, and clue 2 kind of gives it away." (She listed the Quadrilateral Maker and the Square Maker in the parlor, the Kite Maker in the study, the Rectangle Maker in the den, the Trapezoid Maker in the library, the Rhombus Maker in the living room, and the Parallelogram Maker in the game room.)

This student used her knowledge of the Shape Makers to put each Shape Maker character in a room that had a shape she thought it could make. Although she listed correct possible room locations for the Shape Maker characters, her solution seemed to ignore these locations and focus only on clue 2.

Mystery 3, Darla. "Because of clue 2, the Square Maker and the Rectangle Maker are in the study. Because of clue 3, the Rhombus Maker was out of the mansion. I put the Trapezoid Maker in the library, the Parallelogram Maker in the living room, and the Quadrilateral Maker in the game room because I know they can make these shapes. The Kite Maker is the only person left, so the thief was Kaneisha Kite."

Darla did not carefully consider and eliminate all the possibilities. She did not explain why the Kite Maker could not be in the library, living room, or game room.

Mystery 3, Megan. "Clue 2 means that the Square Maker and the Rectangle Maker are in the study. Clue 3 means that the Rhombus Maker was out of the mansion. So the thief was not the Rectangle Maker, Square Maker, or Rhombus Maker. I remember from another student sheet that the Trapezoid Maker can make the shape in the library. The Kite Maker can't make the shapes in the living room or library, but the Parallelogram Maker can. I remember from another sheet that the Quadrilateral Maker can make the shape in the game room. So the thief can't be the Rectangle Maker, Square Maker, Rhombus Maker, Parallelogram Maker, Trapezoid Maker, or Quadrilateral Maker. So it has to be the Kite Maker."

Megan seems to have most of the pieces for a complete argument. The only thing she did not do was say why the Kite Maker could not make the shapes in the library and living room. Also, the order in which she presented her ideas and the second to last statement suggest that she thought she had eliminated all other possibilities, but she hadn't. For instance, she did not say why the Parallelogram Maker could not be in the parlor.

Mystery 3, Juanita and Carlos. Both students used clues 2 and 3 to conclude that the Square Maker and Rectangle Maker were in the study and the Rhombus Maker was out of the mansion. Thus, the Kite Maker could not have been out of the mansion or in the study, so it had to be in the parlor, library, living room, or game room. But of the shapes shown in these rooms, the Kite Maker could only make the square and thus was in the parlor. Juanita said that the Kite Maker could not make the shapes in the library, living room, or game room because it had to have two pairs of adjacent sides equal. Carlos justified this same claim by saying that the Kite Maker had to have a line of symmetry "angle to angle" (meaning vertex to vertex), which none of these shapes have. These students devised different but equally sound proofs.

Mystery 4, Manuel and Toshi. After reading all the clues, Manuel and Toshi conclude that the person in the study is Ricardo Rhombus because of clue 4. They deduce from clue 3 that Kaneisha Kite was out of the mansion at the time of the theft. From clue 2, they derive two suspects—Rectangle Rick and Parallelogram Pete. They then claim that Rectangle Rick must be in the game room—and be the thief—because he couldn't make the shape in the parlor, so Parallelogram Pete had to be in the parlor.

Manuel and Toshi did not explicitly say why Parallelogram Pete had to be in the parlor (hence, their argument was incomplete). So the teacher questioned them about this conclusion: "As Rectangle Rick's lawyer, I don't see how you can say that Parallelogram Pete had to be in the parlor—it might have been Quentin Quadrilateral or Trapezoid Tracy because both of them can make that shape." Manuel and Toshi countered that neither Quentin Quadrilateral nor Trapezoid Tracy could have been in the parlor because neither of them always has two equal sides, which clue 2 said had to be true about the person in the parlor. Thus, although Manuel and Toshi's argument was incomplete, they had made the correct conclusion and were able to satisfactorily rebut challenges to their conclusion.