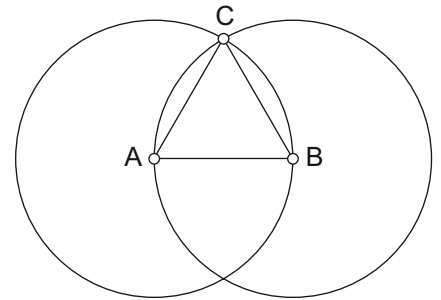


# Euclid's Proposition 1—An Equilateral Triangle

Euclid, a Greek mathematician born around 300 B.C.E., wrote a book called the *Elements*, upon which most school geometry books are still based. All of the geometry in the *Elements* is built up sequentially from a few simple constructions and postulates. Each new property that Euclid presents, or new figure that he constructs, is based on properties he has demonstrated previously. The construction that starts it all is the equilateral triangle. Countless other constructions in the *Elements* depend on being able to construct an equilateral triangle with compass and straightedge. In this activity you'll construct an equilateral triangle using only Sketchpad's freehand tools—the equivalents of Euclid's compass and straightedge.

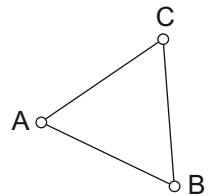
## SKETCH AND INVESTIGATE

1. Construct  $\overline{AB}$ .
2. Construct circle  $AB$ . (Make sure you use point  $A$  for the center and point  $B$  for the radius-defining point.)
3. Construct circle  $BA$ . (Use point  $B$  for the center and point  $A$  for the radius point this time.)
4. Construct  $\overline{AC}$  and  $\overline{CB}$ , where  $C$  is a point of intersection of the two circles.



If you start or finish drawing your segment with the tip of the **Segment** tool directly over the intersection of two objects, both objects will highlight and Sketchpad will construct an endpoint at the intersection.

- Q1** Drag point  $A$  or point  $B$ . What happens to your triangle? Does it appear to stay equilateral?
- Q2** Explain why this triangle is always equilateral. (*Hints:* What roles do the circles play in your construction? How are they related to one another? How are the sides of the triangle related to the circles?)
5. Hide the circles.
  6. Measure the three angles.
- Q3** Drag point  $A$  or point  $B$ . Make a conjecture about the angles in an equilateral triangle.



To measure an angle, select three points, with the vertex your middle selection. Then, in the Measure menu, choose **Angle**.

## EXPLORE MORE

7. Select your entire triangle and, in the Custom Tools menu (the bottom tool in the Toolbox), choose **Create New Tool**. Save the sketch, with an appropriate name, in your own personal folder of tools. (Check with your teacher if you have questions about how or where to save files.) Use your tool on two vertices

## Euclid's Proposition 1—An Equilateral Triangle

continued



of your first triangle to construct a second triangle that shares a side with the first. Keep using the tool on different pairs of vertices and see if you can fill your screen with a tessellation of equilateral triangles. Why do equilateral triangles fill the plane without any gap or overlap?

8. Find a copy of Euclid's *Elements*. Compare his Proposition 1 with the construction you did.