A Folding Investigation

Problem 9 *(Student page 127)* The segment that connects the marked point to the chosen corner is being folded in half, so the crease is along the perpendicular bisector of that segment.

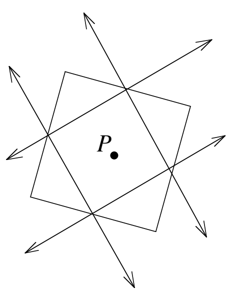
Problem 10 *(Student page 129)* It *is* possible to find an example that is not a hexagon.

Problem 11 *(Student page 129)* Here are some ways to think about these problems:

a. If the lines of the creases were extended beyond the edges of the paper, they would surround a quadrilateral (four lines, four sides). All that the edges of the paper can do is "clip" corners off of that quadrilateral, so the region of paper enclosing P cannot have more than eight sides.

b. Each fold produces a side for the region enclosing P, so there must be at least four sides to the region.

Putting *P* right in the center makes a square inside. Moving *P* will "clip" a corner or two.



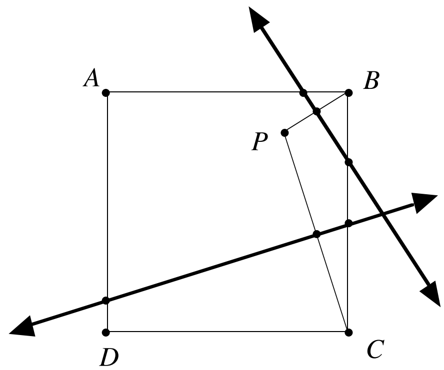
Problem 12 *(Student page 129)* The only way to enclose P within a four-sided figure—the minimum—is to locate P at the center of the square. The region around P is then a square. Moving P pushes one corner of that region off the paper, while pulling the opposite corner onto the paper. We see that actually only two corners of the potential quadrilateral can get "clipped," so six sides is a maximum.

Problem 13 *(Student page 129)* Roughly speaking, moving P from the center towards a corner produces an enclosing region with six sides, and moving P from the center towards the midpoint of a side produces an enclosing region with five sides.

The exact boundaries of the areas can be determined by experiment. Or you can reason it out like this:

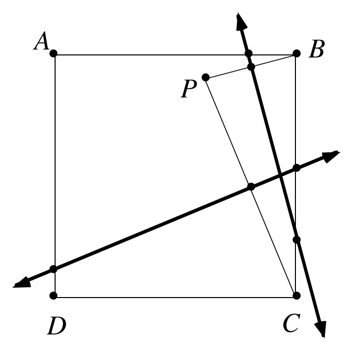
The idea behind this solution comes from Marvin Freedman, a mathematician at Boston University.

You get a new "clipped" side when two creases intersect *outside* the square:



Clipped side

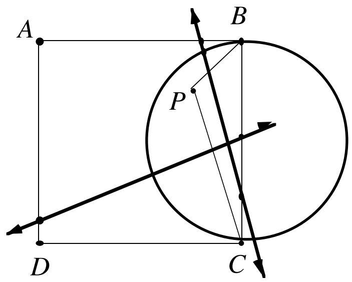
If the creases intersect inside the square, you get no new side:



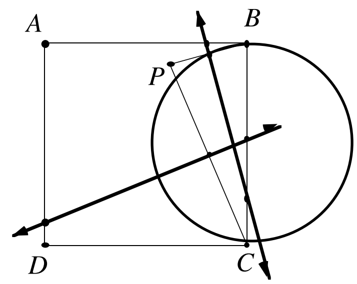
No clipped side

But look at $\triangle\ PBC$ in each of these pictures. The creases are precisely the perpendicular bisectors of sides $\overline{PB}$ and $\overline{PC}$, so you get a new side precisely when these perpendicular bisectors intersect outside the triangle. But *that* happens precisely when $\angle BPC$ is obtuse (why?).

We see that $\angle BPC$ is obtuse if and only if it lies inside the circle whose diameter is $\overline{BC}$:



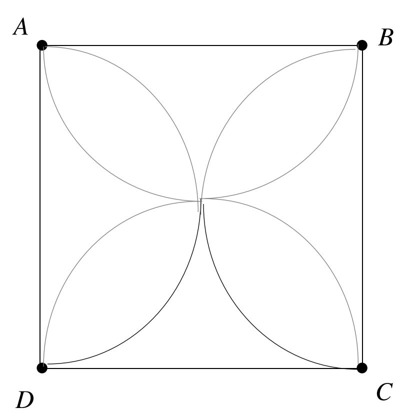
$\angle P$ obtuse



$\angle P$ acute

But it's the same story for every side of the square.

If you think about this a bit, you'll see that the picture below contains some interesting regions.



How many sides do you get if P is in a "leaf" of the rosette? What regions produce a P that leads to a 5-sided polygon?

Problem 14 *(Student page 130)* The software equivalent of creases are the perpendicular bisectors of the four segments connecting point P with the corners of a constructed square.